Good practices
MATH_104BCD_EN

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Description of the problem / exercise: **Geometric probability**

The task belongs to the topic of probability, partly a new task, partly a modification of the B510 PUSE task. Used sets: triangle, square.

Description of the task: construct a shape (target) from basic elements and calculate the probability of hitting a given color with a random shot.

We examine two cases:

I. Using the basic elements of a set we construct a closed shape, which will be the target. Students' task is to calculate the probability of hitting a given color on the target by random shooting.
II. We give the probability of a target's color and the students have to create a closed shape (target board) on which the probability of the given color corresponds to the given values.

Notes on the task:

a) The PUSE B510 task solves the second case in which the area of the fields of different colors are equal, i.e. their probability ratio is 1:1:1:1.

b) Both the triangle and square sets are suitable for constructing a target board. The probability of the colors will be the same if we use the same basic elements of triangles or squares. Because the ratio of the area of the small, medium, large form to the area of the base element is the same in triangle and square as well if we ignore the small square (hole) that is cut out from the corner of the base square.

Case I.

Using the basic elements of a set we construct a closed shape, which will be the target board. What is the probability that we will randomly hit a particular color on the target with a single shot?

a) Creating a target (closed shape) from a set of triangles:

– triangle of 1, 4, 9, 16 basic elements,
– hexagons of 6 or 24 triangles
– rhombus of 2, 8, 18 triangles
– trapezium, parallelogram,
– other shapes, symmetrical or not, e.g. a star of 12 triangles

Creating a target (closed shape) from a set of squares:

– square of 1, 4, 9, 16 basic elements,
– different size rectangles,
– symmetrical shapes, e.g a '+' or a cross shape
– other shapes.

b) Calculating probability:

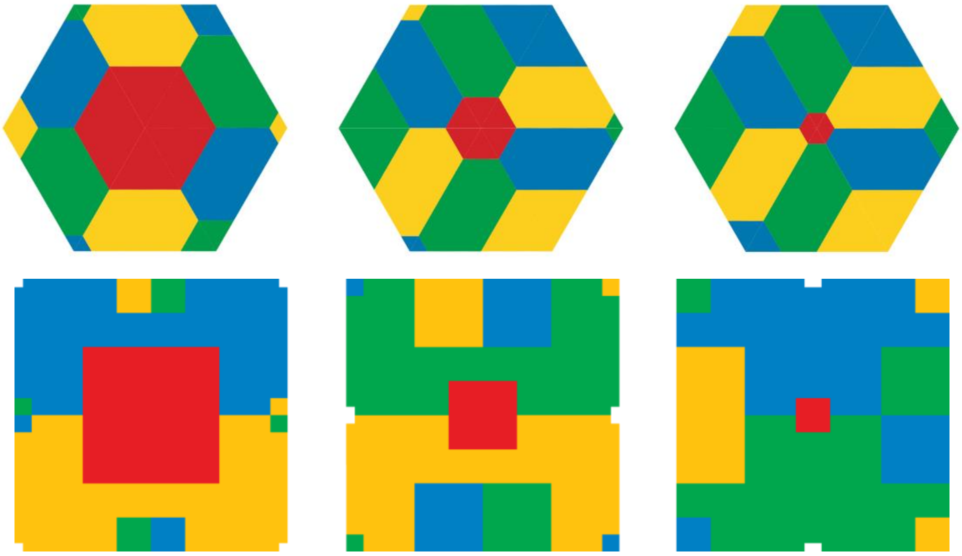
For both triangle and square basic elements (PUSE C 133 task) the ratio of the area

– of the *large* form to the area of the basic element is 1/4,
– of the *medium* form to the area of the basic element is 1/16,
– of the *small* form to the area of the basic element is 1/64,
– of the *hexagon* form to the area of the basic element is 43/64,

For the calculation, let the area of the smallest field of the basic element be 1. Then the area of the small, medium and large field is 4, 16, 43 respectively, and the area of the basic element is 64. If we use *n* basic elements for the target board, the total area is 64*n*.

The probability that we randomly hit a particular color on the target with a single shot is the ratio of the area with the given color to the total area of the target.

- Calculate the probability of hitting a selected color of the target (e.g. the center of the target).



- What is the probability that we will hit the field in the center of the target (red color)?
- What is the probability that we will not hit the selected color?

The area of the total shape (target) is 64*n*, where *n* is the number of basic elements used. The area of the field in the center of the target (red color) for the hexagon target is 6·16, 6·4, 6·1 respectively and for the square target is 4·16, 4·4, 4·1. The probability that we will hit the red field in the center is the ratio of the corresponding area to the total area, which is 1/4, 1/16, 1/64, respectively.

We know that the ratio of the area of the large, medium and small form to the area of the basic element is 1/4, 1/16, 1/64, respectively. This ratio is maintained in the total shape as well, since both the basic element and the form are used equally *n* times in the shape.

- What is the probability of hitting each of the four colors on the target with a single random shot?

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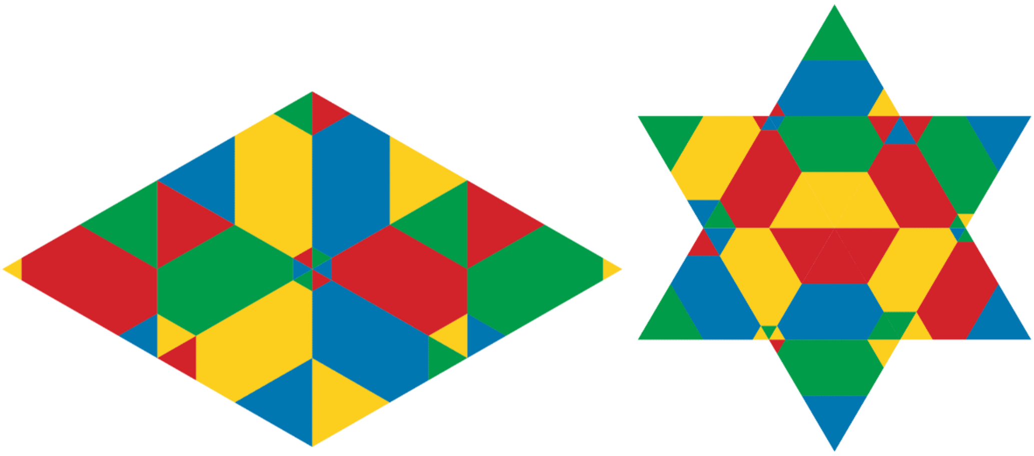

To calculate the probability, create a table in which we write and summarize the size of the area of each colored field based according to the targets:

	base	large	medium	small	total
unit	43	16	4	1	64
1. color R	?	?	?	?	?
2. color Y					
3. color G					
4. color B					

The probability that we randomly hit a particular color on the target with a single shot is the ratio of the area with the given color to the total area of the target.

Case II.

We give the probability of a target's color and the students have to create a closed shape (target board) on which the probability of the given color corresponds to the given values.

- Only the probability of one color is given, the others are not specified. For which values of the probability can a target board be created?
- The probability of hitting the colors is given as the 1:1:1:1 ratio of the area of the colored fields. In this case we are looking for a shape in which the area of the fields of different colors are equal. How many basic elements of a set can be used to construct such a shape? For how many basic elements is it impossible to solve the task that the appropriate target board cannot be created? (PUSE B510 task).
- The probability of hitting the colors is generally given as the $a : b : c : d$ ratio of the area of the colored fields, according to which the target shape must be constructed. For which values and ratios $a : b : c : d$ has the task a solution?

To calculate the probability, we can create a table in which we write down and summarize the size of the area of each colored field.

Let us represent the number of elements according to their color and size with a 4x4 matrix **A**. Let the value of the colored fields on the basic element be a vector ***v*** (43, 16, 4, 1) and the specified ratio is given as a vector ***a***. Then we look for a solution to the equation

$$A \cdot \vec{v} = \vec{a}$$

so that in the rows and columns of the matrix **A** the sum always gives the total number of basic elements.

For the ratio *a* : *b* : *c* : *d* = 1 : 1 : 1 : 1 we have different solutions, where the least number of basic elements is *n* = 3. For *n* = 3 a possible arrangement can be represented with the equation

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 3 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 43 \\ 16 \\ 4 \\ 1 \end{pmatrix} = 48 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

- *Why this exercise is good:* The task develops students' problem solving, logical thinking.
- *Level of teacher training:* According to the degree of difficulty of the task, it can be primary school upper grade, secondary school, teacher training
- *School subject(s):* Mathematics