

Good practices

MATH_110CD_EN

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Description of the problem / exercise: **Cantor & Saxon**

The Saxon's poly-dimensional work (Figure 2) below was created independently of the Cantor set, in this exercise we examine the work and find geometric and mathematical connections with classical fractals.

The image is made up very clearly by the possible permutation of halving the diagonals of the square:

In order to get an idea of immaterialization, we may set up a logical experiment: If there is a set of planes made up by at least two other sets of planes that in turn include two further sets of planes each, and so forth ad infinitum, then we may witness the termination of the plane as a form, as it becomes a set of points. If, on the other hand, we take space, then the same process leads to the depletion of space or an object, and the substance, after reaching a density of infinite fineness, is immaterialized, is transformed in our mind definitively. The direction of progress is inward-building (interior = taking some of the area away, leaving a gap), that is, diminution of the plane, since the aim is to decompose the form. We mark the corners as connecting points again, in each of which we leave the smaller black squares obtained from the 1:3 proportion of the sides of the previous scale. We follow the same procedure several times. It is obvious that in the first square there are four smaller $T_1 = 4/9$ elements left, and four more elements in each of them, up to infinity... In the meantime the area of the starting square ($T_0 = 1$) has been diminished $T_3 = 1 - (5/9) - (5/9) \times (4/9) - (5/9) \times [(4/9) \times (4/9)] = 0.087792...$ times in three steps, while the number of squares will be $D_3 = 64$. Further number of squares can be calculated by the formula $D_n = 4^n$ and provided $n = \infty$, the remaining form will be a cloud of dust made up of infinitesimal granules, invisible to the naked eye. In this fierce fight our black square has whitened, losing the last bits of its area (Figure 1).

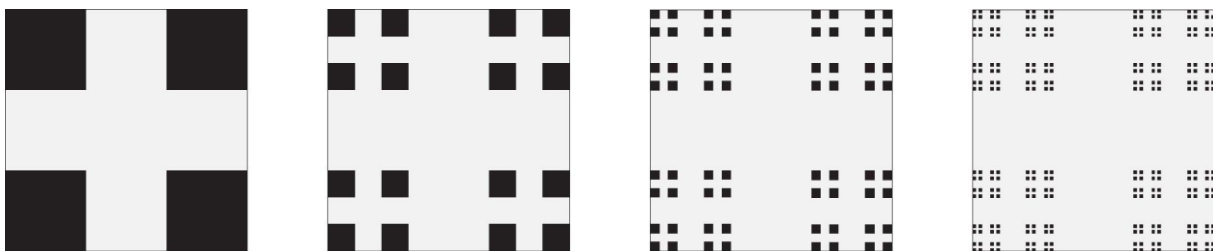


Figure 1: A didactic illustration of SAXON's artwork

This complete transfiguration, this absolutely transparent state, Saxon could only model in painting by using such elements as even in themselves represent the supremacy of pure sensation. Thus two basic suprematism elements, the square and the cross through which the square is divided into four parts (see Kasimir Malevich), have served as points of departure. In this case, the square bears a yellow color symbolizing existence, whereas its opposite, the cross is characterized by a white tone

that creates an impression of emptiness. He must mention that to me the yellow color in relation to white reflects the sensations of being and non-being, something and nothing, in a more vivid contrast than, say, black and white would do. During the construction of the picture, i.e. the deconstruction of the yellow square, I came to sense total depletion, or, more precisely, to set up a poly-dimensional net. The net that connects micro- and macro-worlds, is the virtualization of the absolute mind, which, stretched in infinite dimension structures as a hyper-filter, incessantly attempts to jettison the imperfect objects (yellow squares) of existence from its 'body' (Figure 2).

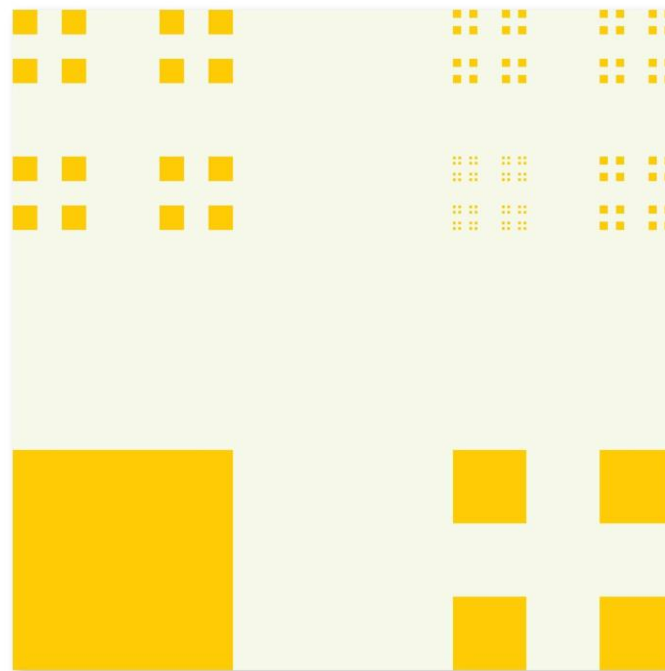


Figure 2: SAXON, Immaterial transit 1997, oil on wood 152x152 cm

Sets with similar properties are called Cantor sets. The location, length and number of omitted intervals are variable.

Let be given an interval of real numbers. In the first step, we remove from it a finite number of disjoint open intervals, but at least one, leaving at least 2 closed intervals, but a finite number of closed intervals.

In the second step, we again remove a finite number of open intervals from each small remaining interval.

Iterating this, after an infinite number of steps, we are left with the points that did not belong to any of the intervals removed.



Figure 3: Cantor's classic fractal image / https://en.wikipedia.org/wiki/Cantor_set

It can be shown that the resulting Cantor sets are homeomorphic and have continuum numerosity, i.e. they have the same numerosity as real numbers. By choosing the ratio of the omitted intervals to the remaining intervals, for any given number $[0,1]$ there is a Cantor set of dimension $[0,1]$.

The Smith-Volterra-Cantor set, like the Saxon conformation above, is constructed by removing the middle third of each interval in the n th step instead of the middle $1/(2^n+2)$ (i.e., a $1/2^{2n}$ long interval). The total length of the omitted intervals is

$$\sum_{n=0}^{\infty} 2^n (1/2^{2n+2}) = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots = \frac{1}{2}$$

so that the length of the Smith-Volterra-Cantor set is $1/2$. Despite the positive Lebesgue measure, even this set is nowhere dense.

The Cantor dust is a higher dimensional version of the Cantor set. These dusts are finite Descartes effects of the Cantor set. Like the Cantor set, Cantor dust has zero dimension.

- *Why this exercise is good:* Interdisciplinary approach. The relationship between fine arts and mathematics. Relationships between art history and the history of mathematics.
- *Which level is recommended:* Secondary school (advanced level), mathematics teacher training
- *School subject(s):* Art & mathematics
- *Comments:* See more Saxon's Footless Chair (1998)

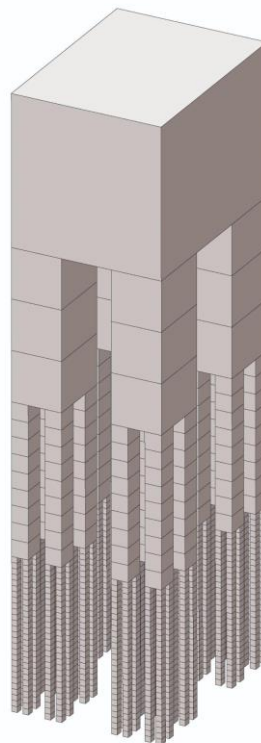


Figure 4: Saxon's Footless Chair (1998) / <http://www.saxon-szasz.hu/writings/by-saxon/footless-chair/>