Good practices
MATH_113BCD_EN

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Description of the problem / exercise: **Rosettes and Poly-Universe**

The *Rosette (rose window)* group is a translation-free group, its name comes from church windows. There are infinitely many rosette groups, which can be classified into two substantially different subgroups. One of them consists of groups of rotations about a single point: rotations with integer multiples of 2π/n. They belong to the cyclic rosette group, written C_n. These groups do not have reflection symmetry. Besides the mentioned rotations, if the group also have *n* reflection symmetries with axes going through the centre of rotation, they belong to the dihedral group, written D_{2n}.

Figure[1]: Left St Peter and Paul church Gorlitz, C_6 rosette group. Right: Cambridge, Cambridgeshire, England, UK, photo by Leo Reynolds, D_{10} rosette group

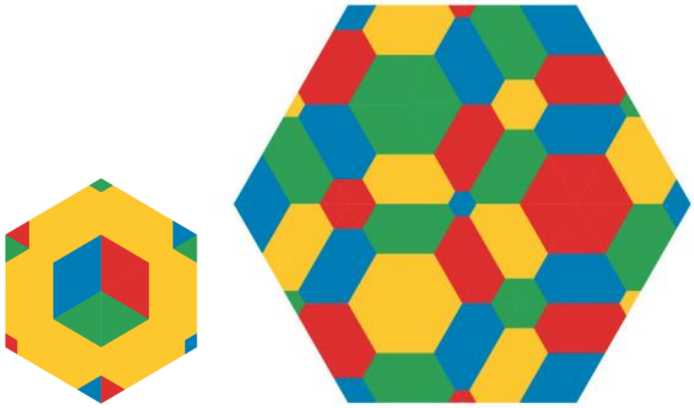
Rosettes are not simply on buildings, we may find plenty of them in nature: when we cut an apple or orange in half; or when we marvel at a lovely flower or cactus, and we may even find starfish from the D_{10} symmetry group. Among old door locks and manhole covers, we can discover rosette symmetry; what's more, looking at our cars' hubcaps, we can quickly find their 'rosette code', and we could continue for long.

Rosettes and the Poly-Universe

The book contains many examples of rosettes assembled by triangle and square elements. We can arrange 6 triangles to make up a hexagon with D_6 rosette symmetry. Examples of this can be seen on task sheets 125_B and 212_B.
Using the entire set, large hexagons assembled by triangle connections of the same colour and size have D_6 symmetry, see Tasks 130_BC and 212_B. The picture on the right in the following figure is an example of this. Task 212_B from the Combinatorics chapter asks for the number of constructions

[1] Sources of pictures:
https://commons.wikimedia.org/wiki/File:St_Peter_and_Paul_church_Gorlitz_round_window.jpg
https://www.flickr.com/photos/lwr/6333291955/ (2020.02.06)

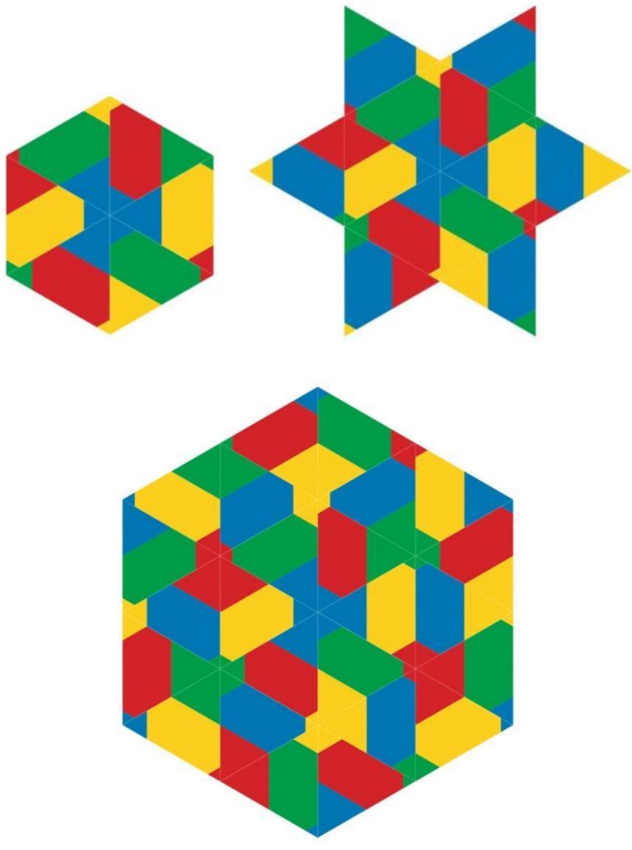
with the same size and colour connections. There are 12 ways to construct smaller (of 6 triangles) and larger (of 24 triangles) hexagons as well with the given conditions. We can establish that each construction has D_6 rosette symmetry.



Rosettes from triangles on PUSE task sheets.

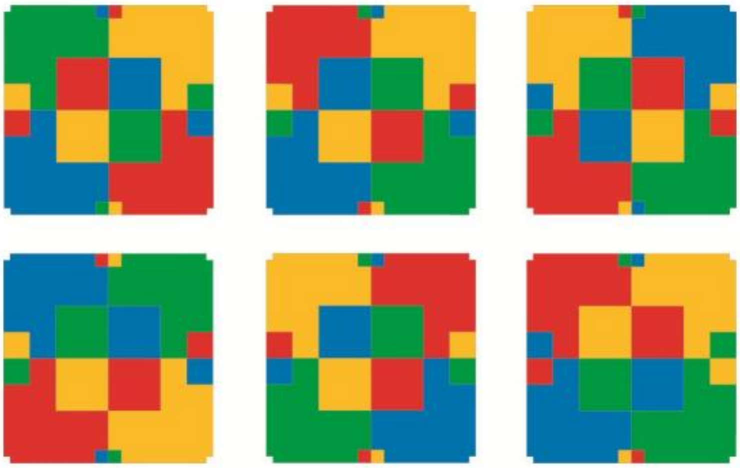
Flipping through the book we notice that the shapes we construct from triangles can either have no symmetries or have dihedral symmetry (see Figure above). Would it be possible to construct rosettes with cyclic symmetry, using triangles? The answer is yes, but to keep the restriction of total side connection, we can only work with connections of the same colour and give up on same size connections.

See Figure below for examples, all three of them are C_3 cyclic rosette groups.



Cyclic rosette groups from triangle elements.

Nice rosettes can be constructed from square elements as well. First, let's examine the task sheets from this perspective. Many questions could be raised about symmetry in connection with Task 108_A; now the given solutions do not contain any symmetries at all. One possible solution of 231_C on the teacher sheet has D_4 dihedral symmetry.



Rosettes from square elements on PUSE task sheets.

In the case of squares also, we can ask whether it is possible to construct a cyclic group. The answer is yes, two examples from Task 203_A represent the C_2 cyclic group.

Cyclic rosette groups from square elements

We can also ask whether it is possible to construct bigger shapes (using more than four squares) of rosette symmetries.

Finally, on rosettes, we need to mention Task 508_AB which encourages group work constructions, using all three sets. For this, one solution on the teacher sheet is also a D_6 dihedral symmetry rosette.

- *Why this exercise is good:* Symmetry is one of the ordering principles of nature. Symmetry, asymmetry, and dissymmetry (minor damage to symmetry) are all present in the body structure of plants, animals, and humans. Although, symmetry is still felt to be dominant. We can also discover symmetry in the laws of physics, in the crystal structure of materials. Symmetrical objects give us a sense of harmony, of order, and therefore symmetry is reflected in human creations, in our everyday objects, and artworks. We can have a real interdisciplinary (STEAM) session if we look for examples of rosette symmetries in nature, in different sciences, in art, in our everyday life.
- *Which level is recommended:* Upper primary, secondary school, teacher training

- *School subject(s):* Mathematics, arts
- *Comments:* References

Rosette from mixed Poly-Universe elements.

- Bérczi, S. (1986) Escherian and non-Escherian developments of new frieze types in Hanti and old Hungarian communal art, *MC Escher: Art and Science*, 349-358.
- Darvas, Gy. (1999) Szimmetria a tudományban és a művészetben, *Magyar Tudomány*, 3, [in Hungarian] Utánközlés: Retrieved from http://members.iif.hu/visontay/ponticulus/rovatok/hidverok/szimmetria_darvas.html (2020. 02. 06.)
- Hargittai I. & Lengyel, Gy. (2003) A hét egydimenziós szimmetria-tércsoport magyar hímzéseken, [in Hungarian]. Retrieved from http://members.iif.hu/visontay/ponticulus/rovatok/hidverok/hargittai2.html (2020. 02. 06.)
- Szász SAXON, J., Stettner, E., eds. (2019) *PUSE (Poly-Universe in School Education) METHODOLOGY – Visual Experience Based Mathematics Education*, Szokolya: Poly-Universe Ltd. (Publisher: Zs. Dárdai), [open access in pdf from http://poly-universe.com/puse-methodology/ 254 p. ISBN 978-615-81267-1-7].