Good practices
MATH_114BCD_EN

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Description of the problem / exercise: **Frieze symmetries and Poly-Universe**

The class of isometry groups containing a single translation are *frieze groups*.

There are exactly seven ways of creating (infinite) linear patterns (friezes) which are generated by the (infinite) repetition of one motif. The codes and patterns of these subgroups (represented by letters) are shown in the Table. The explanation for codes is the following:

1. Letter *p* at position 1 comes from the word pattern.
2. At position 2 there is a letter *m* if the pattern contains reflection symmetry with an axis perpendicular to the direction of translation. Otherwise, we write *1*.
3. At position 3 there are letters *m* or *a* if the pattern contains reflection symmetry with an axis parallel to the direction of translation, or a glide reflection, respectively.
4. At position 4 comes the order of rotation. It can be proven that frieze patterns can only have rotation centres of order two.

Group code	Pattern
p111	...LLL...
p112	...NNN...
p1m1	...DDD...
p1a1	...bpbpbp..
pm11	...AAA...
pmm2	...HHH...
pma2	...ΛVΛ...

Classification of frieze symmetries

Each of these frieze symmetries also appears in the Hungarian decorative art from the era of the Hungarian conquest of the Carpathian basin (Bérczi, S. 1986), but similarly can be found in other nations' art. We can learn about it in *Eurasian art* collection edited by Szaniszló Bérczi, which can be downloaded from this website: http://www.federatio.org/tkte.html. We come across friezes everywhere: when walking along an old street if we look up to the ornamentation of the buildings

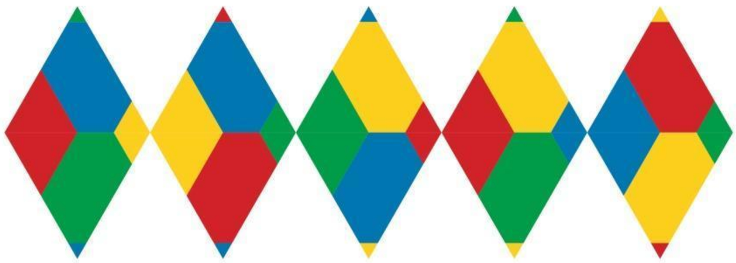
– or look down, noticing the border of an ornate floor or panelling. Each of the seven frieze patterns appears on Hungarian cross-stitch embroidery as we read the study of István Hargittai and Györgyi Lengyel. (Hargittai I. & Lengyel, Gy. 2003).

Friezes and the Poly-Universe

As we can see on figures below, Poly-Universe constructions allow for all seven-frieze symmetries. Those patterns which have reflection symmetry with an axis parallel to the direction of translation can only be assembled by arranging the elements at least into two lines (because Poly-Universe elements do not have inner isometries).

p111

p112



p1m1

p1a1

pm11

pmm2

pma2

Now let’s see which tasks of the book can be extended by questions on frieze symmetry.

Task 104_A asks to build quadrilaterals by assembling triangle elements. We can establish that an odd number of triangles make up trapezia, with an even number we get parallelograms. This task can be extended by asking the following question: with connections of the same colour and size, what type of symmetry does the constructed linear pattern have? The answer: frieze pattern with p1a1 symmetry.

This extension may be applied to other tasks, e.g. 106_A, 130_BC. At 306_A, the task is to identify the rules of the connected triangle elements. Apart from the many rules, it is worth adding symmetry rules like p111 and pm11 frieze patterns.

p111

p112

p1m1

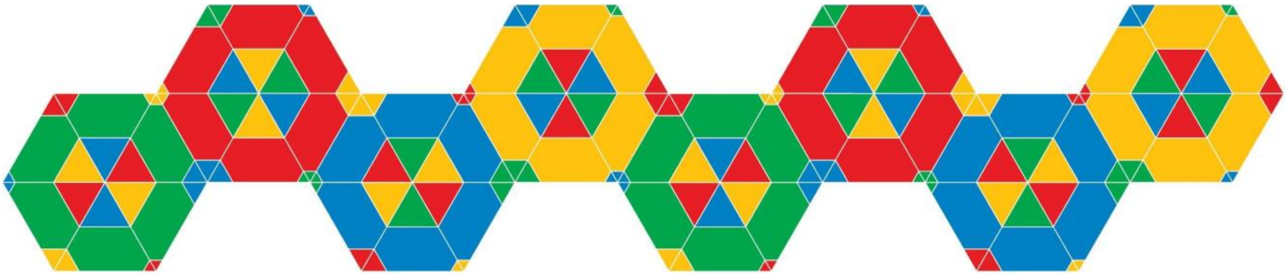
p1a1

pm11

pmm2

pma2

Finally, the most beautiful frieze pattern from Poly-Universe elements in the book is the decorative frieze at the beginning of chapters, constructed by János Szász Saxon. This p1a1 frieze contains glide reflection.



p1a1 frieze, decorating the PUSE book

- *Why this exercise is good:* Symmetry is one of the ordering principles of nature. Symmetry, asymmetry, and dissymmetry (minor damage to symmetry) are all present in the body structure of plants, animals, and humans. Although, symmetry is still felt to be dominant. We can also discover symmetry in the laws of physics, in the crystal structure of materials. Symmetrical objects give us a sense of harmony, of order, and therefore symmetry is reflected in human creations, in our everyday objects, and artworks. We can have a real interdisciplinary (STEAM) session if we look for examples of frieze symmetries in nature, in different sciences, in art, in our everyday life.
- *Which level is recommended:* Primary upper grades, secondary school, teacher training
- *School subject(s)*: Mathematics, IT, arts
- *Comments*: References

  - Bérczi, S. (1986) Escherian and non-Escherian developments of new frieze types in Hanti and old Hungarian communal art, MC Escher: Art and Science, 349-358.
  - Darvas, Gy. (1999) Szimmetria a tudományban és a művészetben, Magyar Tudomány, 3, [in Hungarian] Utánközlés: Retrieved from *http://members.iif.hu/visontay/ponticulus/rovatok/hidverok/szimmetria_darvas.html* (2020. 02. 06.)
  - Hargittai I. & Lengyel, Gy. (2003) A hét egydimenziós szimmetria-tércsoport magyar hímzéseken, [in Hungarian]. Retrieved from *http://members.iif.hu/visontay/ponticulus/rovatok/hidverok/hargittai2.html* (2020. 02. 06.)
  - Szász SAXON, J., Stettner, E., eds. (2019) PUSE (Poly-Universe in School Education) METHODOLOGY – Visual Experience Based Mathematics Education, Szokolya: Poly-Universe Ltd. (Publisher: Zs. Dárdai), [open access in pdf from *http://poly-universe.com/puse-methodology/* 254 p. ISBN 978-615-81267-1-7].